Mock board Exam

MATHEMATICS

CLASS XII

Time allowed : **3** hours   Maximum Marks : **100**

General Instructions:

(i) ***All*** *questions are compulsory.*
(ii) *This question paper contains* ***4*** *printed pages and* ***29*** *questions.*
(iii) *Question* ***1- 4*** *in* ***Section A*** *are very short-answer type questions carrying* ***1*** *mark each.*
(iv) *Question* ***5-12*** *in* ***Section B*** *are short-answer type questions carrying* ***2*** *marks each.*
(v) *Question* ***13-23*** *in* ***Section C*** *are long-answer-**I** type questions carrying* ***4*** *marks each.*
(vi) *Question* ***24-29*** *in* ***Section D*** *are long-answer-**II** type questions carrying* ***6*** *marks each.*

Section-A

Questions 1 to 4 carries 1 mark each.

1. If $f(x) = \left(25 - x^4\right)^{1/4}$ for $0 < x < \sqrt{5}$, then find $f\left(f\left(\frac{1}{2}\right)\right)$.

2. Find the value of p, such that the matrix $\begin{bmatrix} -1 & 2 \\ 4 & p \end{bmatrix}$ is singular.

3. Write all the unit vectors in XZ-plane.

4. Let * be a binary operation, defined by a*b = 3a + 4b – 2, find 4*5.

Section-B

Questions 5 to 12 carry 2 marks each.

5. Write the anti-derivative of, $a > 0$ by using inspection method.

6. If $adjA = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$ then find the possible value of |A|.

7. If $f(x) = \sqrt{\cot^2(x^2)+1}$ then, find $f'\left(\frac{\sqrt{\pi}}{2}\right)$.

8. Write the value of θ if, $\tan^{-1}(2) + \tan^{-1}(3) + \theta = \pi$.

9. If *x* changes from 4 to 4.01, then find the approximate change in log x .

10. Obtain the differential equation of the family of circles passing through the points (*a*, 0) and (-*a*, 0).

11. If $|\vec{a}| = a$, then find the value of $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2$.

12. *If* $P(A) = \frac{3}{5}$, $P(B) = \frac{2}{5}$ *and* $P(A \cap B) = \frac{1}{5}$ *find* $P\left(\frac{\bar{A}}{\bar{B}}\right)$.

Section-C

Questions 13 to 23 carry 4 marks each.

13. If A = diag [a b c], where a, b, c are non-zero, find A^{-1}.

14. Let $f(x) = \begin{cases} x^2 \left|\sin\frac{\pi}{x}\right| & x \neq 0 \\ 0 & x = 0 \end{cases}$ $\forall\, x \in R$, then show that $f(x)$ is differential at $x = 0$ but not differentiable at x = 2.

OR

Find all the point of discontinuity of the function *f* defined by **x**.

15. Find $\frac{d^2x}{d\theta^2}, \frac{d^2y}{d\theta^2}$ *and* $\frac{d^2y}{dx^2}$ *at* $\theta = \frac{\pi}{4}$ if $y = a\ \sin\theta$ and $x = a\ (5 - \cos\theta)$.

16. Show that the area of the triangle form by the tangent and the normal at the point (*a*, *a*) on the curve $y^2(2a - x) = x^3$ and the line x=2*a*, is $\frac{5a^2}{4}$ *sq.units* .Write any value that you like the most.

OR

If $C = 0.003x^3 + 0.02x^2 + 6x + 250$ gives the amount of carbon pollution in the air in the area on the entry of x number of vehicles, then find the marginal carbon pollution in the air, when three vehicles have entered in the area and write which value does the question indicate.

17. A window of fixed perimeter (including the base of the arc) is in the form of a rectangle surmounted by a semi-circle. The semicircular portion is filled with colored glass while the rectangular part is filled with clear glass. The clear glass transmits three times as much light per sq. meter as the coloured glass does. What is the ratio of the sides of the rectangle so that the window transmits the maximum light?

18. Evaluate $\int_0^{\frac{\pi}{2}} \log(\tan x + \cot x)\, dx$

19. Solve the differential equation: $\frac{dy}{dx} - \frac{1}{x}.y = 2x^2$.

OR

Solve: $(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$

20. $\vec{a}, \vec{b}, \vec{c}$ are the unit vectors. Suppose $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

21. Find the values of ***a*** so that the following lines are skew

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-a}{4}, \frac{x-4}{5} = \frac{y-1}{2} = z.$$

22. In a game, a man wins Rs.1Lakh for a one and loses a Rs.50, 000 for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a one. Find the expected value of the amount he wins/loses.

23. From a pack of 52 playing cards, a card is accidently dropped. From the remaining 51 cards, two cards are drawn at random (without replacement) and are found to be both spades. Find the probability that the dropped card was a card of the club?

Section-D

Questions 24 to 29 carry 6 marks each.

24. Evaluate $\int \sqrt[3]{\tan x}\, dx$.

OR

Evaluate $\int_1^4 (x^2 - x)dx$ as the limit of a sum.

25. Let $A = N \times N$. Let * be a binary operation on A defined by $(a,b)*(c,d) = (ad + bc, bd)$. Then

(i) find the identity element of $(A,*)$

(ii) is $(A,*)$ commutative?

26. Using integration find the area of the region included between the curves $y = x^2 + 1, y = x, x = 0 \text{ and } y = 2.$

OR

Make the rough sketch of the region given below and find its area using integration: $\{(x, y): 0 \le y \le x^2 + 3;\ y \ge 2x + 3; 0 \le x \le 3\}.$

27. Find the equation of the line of intersection of planes $4x + 4y - 5z = 12$ and $8x + 12y - 13z = 32$ in the vector and symmetric form.

28. If $p \neq 0, q \neq 0, \begin{vmatrix} p & q & p\propto+q \\ q & r & q\propto+r \\ p\propto+q & q\propto+r & 0 \end{vmatrix}=0$, then, using properties of determinants, prove that at least one of the following statements is true:

(a) p, q, r are in G.P.

(b) $\propto$ is the root of the equation $px^2 + 2qx + r = 0$.

OR

For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

Show that $A^3 - 6A^2 + 5A + 11\,I = O$. Hence, find A^{-1}.

29. There are two factories, one located at Vidyut Nagar and the other in Delhi, from these locations, a certain number of machines are to be delivered to each of the three depots situated at P, Q and R. The weekly requirements of the depots are respectively 5, 5, and 4 units of the machines while the production capacity of the factories at Vidyut Nagar and Delhi are 8 and 6 units respectively. The cost of transportation per unit is given below.

From ↓ / To →	Cost (in Rs.)		
	P	Q	R
Vidyut Nagar	160	100	150

Delhi	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum?
What does the minimum transportation cost?

Marking Scheme as per CBSE

Mock Exam-2016-17

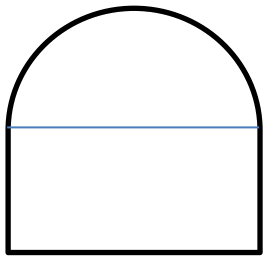
Class XII

S.No.	View Points	Mark
	Section-A	
1.	$fof\left(\frac{1}{2}\right)=\left(25-\left(25-\left(\frac{1}{2}\right)^4\right)\right)^{1/4}=\frac{1}{2}$	**1**
2.	**P=-8**	**1**
3.	$\cos\theta\ \hat{i}+\sin\theta\ \hat{k}$	**1**
4.	4*5 = 3.4 + 4.5 – 2 **= 30**	**½+ ½**
	Section-B	

5.	$\frac{d}{dx}\left(\frac{\tan^{-1}a^x}{\log a}\right)=\frac{a^x}{1+a^{2x}}$ $\therefore by\ \ inspection\ method\ \ antiderivtive\ of\ \ \frac{a^x}{1+a^{2x}}\ is\ \frac{\tan^{-1}a^x}{\log a}.$	**1** **1**
6.	$\|A\|=\pm\|\text{adj}A\|^{1/2}$ $\|A\|=\pm4^{1/2}=\pm2$	**1** **1**
7.	$f(x)=\text{cosec}\,x^2$ $f'(x)=-2x\,\text{cosec}\,x^2\cot x^2$ $f'\left(\frac{\sqrt{\pi}}{2}\right)=-2\left(\frac{\sqrt{\pi}}{2}\right)\sqrt{2}=-\sqrt{2\pi}$	**½** **½** **1**
8.	$\tan^{-1}(2)+\tan^{-1}(3)+\theta=\pi.$ $\pi+\tan^{-1}\frac{2+3}{1-2.3}+\theta=\pi$ $\tan^{-1}(-1)+\theta=0$ $\theta=\frac{\pi}{4}$	**½** **½** **½** **½**
9.	Let $y=\log_e x, x=4, \delta x=.01$ $\frac{dy}{dx}=\frac{1}{x}$ $dy=\left(\frac{dy}{dx}\right)_{x=4}\times\delta x=\frac{1}{400}=.0025$	**½** **½** **1**
10.	$x^2+(y-b)^2=a^2+b^2 or, x^2+y^2-2by=a^2$.........(1) $2x+2y\frac{dy}{dx}-2b\frac{dy}{dx}=0\Rightarrow 2b=\frac{2x+2y\frac{dy}{dx}}{\frac{dy}{dx}}$.........(2) Substituting in (1), $(x^2-y^2-a^2)\frac{dy}{dx}-2xy=0$	**½** **1** **1/2**

11.	$= a^2(\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$ $= a^2\left(3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)\right)$ $= a^2(3-1) = 2a^2$	½ **1** **1/2**
12.	$P(\bar{A}/\bar{B}) = \dfrac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$ $= \dfrac{1-P(A\cup B)}{1-P(B)}$ $= \dfrac{1-[P(A)+P(B)-P(A\cap B)]}{1-P(B)}$ **=1/3**	½ ½ ½ **1/2**
	Section-C	
13.	**$\lvert A\rvert = abc \neq 0$** **$A_{11}=bc$ $A_{21}=0$ $A_{31}=0$** **$A_{12}=0$ $A_{22}=ac$ $A_{32}=0$** **$A_{13}=0$ $A_{23}=0$ $A_{33}=ab$** **adjA=** $\begin{bmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{bmatrix}$ $A^{-1}=\dfrac{adjA}{\lvert A\rvert}=\dfrac{1}{abc}\begin{bmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{bmatrix}$ **$A^{-1}=diag[a^{-1}\ b^{-1}\ c^{-1}]$**	½ **1+1/2** **1** ½ **1/2**
14.	$f(x) = \begin{cases} x^2\left\lvert\sin\dfrac{\pi}{x}\right\rvert & x\neq 0 \\ 0 & x=0 \end{cases} \quad \forall\ x\in R,$ at x = 0 LHD = $\lim\limits_{h\to 0}\dfrac{f(0-h)-f(0)}{-h} = \lim\limits_{h\to 0}\dfrac{h^2\left\lvert\sin\dfrac{\pi}{-h}\right\rvert - 0}{-h} = 0$ $RHD = \lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h} = \lim\limits_{h\to 0}\dfrac{h^2\left\lvert\sin\dfrac{\pi}{h}\right\rvert - 0}{h} = 0$ $at\ x = 2$	 **1** **1**

	$\text{LHD} = \lim_{h\to 0}\frac{f(2-h)-f(0)}{-h} = \lim_{h\to 0}\frac{(2-h)^2\left\|\sin\frac{\pi}{2-h}\right\|-0}{-h} = -\infty$	**1**
	$RHD = \lim_{h\to 0}\frac{f(2+h)-f(0)}{h} = \lim_{h\to 0}\frac{(2+h)^2\left\|\sin\frac{\pi}{2+h}\right\|-0}{h} = \infty$ **Clearly f is differentiable at 0 but not at x = 2** **OR** $f(x)=\begin{cases} \|x\|+3, & if \quad x\le -3 \\ -2x, & if \quad -3<x<3 \\ 6x+2 & if \quad x\ge 3\end{cases}$	**1**
	$\because$ given function is modulus function and polynomial function which always continuous everywhere, so we have only two doubtful points for discontinuity at x = -3 and x = 3.	1/2
	LHL=$f(-3-0)=\lim_{h\to 0} f(-3-\text{h})=\lim_{h\to 0}\{\|(-3-\text{h})\|+3\}=6$	½
	$RHS = f(-3+0)=\lim_{h\to 0} f(-3+\text{h})=\lim_{h\to 0} -2(-3+\text{h})=6$	½
	$f(-3)=\|-3\|+3=6.$	½
	LHL=$f(3-0)=\lim_{h\to 0} f(3-\text{h})=\lim_{h\to 0}-2(3-\text{h})=-6$	½
	$RHS = f(3+0)=\lim_{h\to 0} f(3+\text{h})=\lim_{h\to 0} 6(3+\text{h})+2=20$	½
	$f(3)=3.6+2=20.$	½
	Clearly f(x) is continuous everywhere except x = 3.	1/2
15.	$\frac{dx}{d\theta}=a\sin\theta \qquad \frac{dy}{d\theta}=a\cos\theta=\frac{a}{\sqrt{2}}$	**1**
	$\frac{d^2x}{d\theta^2}=a\cos\theta \qquad \frac{d^2y}{d\theta^2}=-a\sin\theta=-\frac{a}{\sqrt{2}}$	**1**
	$\frac{dy}{dx}=\cot\theta \qquad \frac{d^2y}{dx^2}=-\text{cosec}^2\theta.\frac{1}{a\sin\theta}=-\frac{\text{cosec}^3\theta}{a}=-\frac{2\sqrt{2}}{a}$	**2**

16.	$y^2(2a-x)=x^3$ $2yy'(2a-x)-y^2=3x^2$ $y'=\dfrac{3x^2+y^2}{2y(2a-x)}=\dfrac{4a^2}{2a^2}=2$ $Eq\,of$ tangent and normal $y-a=2(x-a)$ $y-a=-\dfrac{1}{2}(x-a)$ $Now\,area\,of\,three\,lines$ $y-a=2(x-a)$ $y-a=-\dfrac{1}{2}(x-a)\,and\;\;x=2a$ $Finding$ Points $area=\dfrac{5a^2}{2}\,sq.unit.$ $Any\;justified\;value$ **OR** $C=0.003x^3+0.02x^2+6x+250$ $C'=0.009x^2+0.04x+6$ $C'(3)=0.009\times3^2+0.04\times3+6=6.201$ Any justified value	½ ½ ½ ½ ½ ½ 1 1 1 1 1
17.	2x+2x+2y+πx = k(constant) A = ¾(area of rectangle)+ ¼ (area of semi-circular part) $=3/4(2xy)+1/4\left(\dfrac{1}{2}\pi x^2\right)$ $=\dfrac{3}{4}x(k-4x-\pi x)+\dfrac{\pi}{8}x^2$ $=\dfrac{3}{4}(kx-4x^2-\pi x^2)+\dfrac{\pi}{8}x^2$ 	**Fig-1** 1 1/2

	$Now\frac{dA}{dx}=\frac{3}{4}\left(k-8x-2\pi x\right)+\frac{\pi}{4}$ $for\min or\max\frac{dA}{dx}=0$ $\frac{3}{4}\left(k-8x-2\pi x\right)+\frac{\pi}{4}x=0$ $\frac{3}{\not 4}\left(k-8x-2\pi x\right)=-\frac{\pi}{\not 4}x$ $3\left(k-8x-2\pi x\right)=-\pi x$ $3k=24x+6\pi x-\pi x=24x+5\pi x$ $x=\frac{3k}{24+5\pi}$ $Now\frac{d^2A}{dx^2}=\frac{3}{4}\left(0-8-2\pi\right)+\frac{\pi}{4}<0$ ∴ Maximum light will enter through the window if $x=\frac{3k}{24+5\pi}\ and\ y=\frac{12k+2\pi k}{24+5\pi}$ $2x:y=3:\pi+6$	1 1 1 1/2

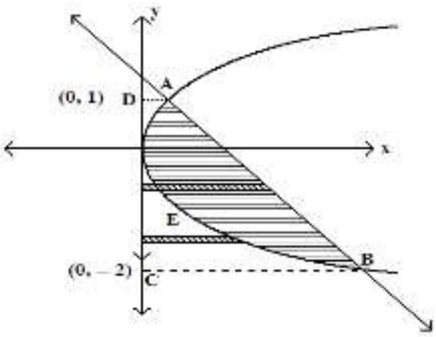
18.	$Let\ \ I=\int_0^{\frac{\pi}{2}}\log(\tan x+\cot x)\ dx\ \ ==\int_0^{\frac{\pi}{2}}\log 2-\log\ \sin 2\text{x}\ dx=\frac{\pi}{2}\log 2-I_1$ $Now\ I_1=\int_0^{\frac{\pi}{2}}\log\ \sin 2\text{x}\ dx$ $putting\ \ 2x=t\ \ dx=dt/2$ $I_1=\frac{1}{2}\int_0^{\pi/2}\log\ \sin\text{t}\ dt----\text{(i)}$ $2I_1=\int_0^{\pi/2}\log\ \sin\left(\frac{\pi}{2}-\text{t}\right)dt$ $=\int_0^{\pi/2}\log\ \cos\text{t}\ dt----\text{(ii)}\Rightarrow 2\text{I}_1=\int_0^{\pi/2}[\log\ \sin 2\text{t}-\log\ 2]\ dt$ $2I_1=\int_0^{\pi/2}\log\ \sin 2\text{t}\ dt-\frac{\pi}{2}\log 2$ $I_1=-\frac{\pi}{2}\log 2$ $\therefore I=\int_0^{\frac{\pi}{2}}\log(\tan x+\cot x)\ dx=\frac{\pi}{2}\log 2-\left(-\frac{\pi}{2}\log 2\right)=\pi\log 2$	**1** **1** **1** **1**
19.	$\frac{dy}{dx}-\frac{1}{x}\ \ y=2x^2$ $\text{I. F.}=e^{\int-\frac{1}{x}dx}=e^{-\log x}=\frac{1}{x}$ Sol. of diff eq^{n} $y\ \ \text{I. F.}=\int Q\ \ \ \ \text{I.F.}\ dx$ $y\ .\frac{1}{x}=\int 2x^2\times\frac{1}{x}\ dx+c$ $=2\int x\ \ dx+2$ $=2\frac{x^2}{2}+c$ $y/x=x^2+c$ $y=x^3+cx$ will be required sol. of given diff eq^{n}. **OR**	**1** **1** **1** **1**

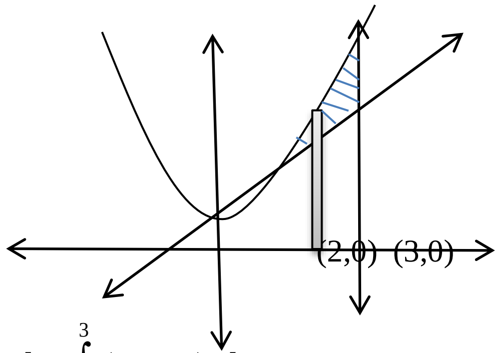
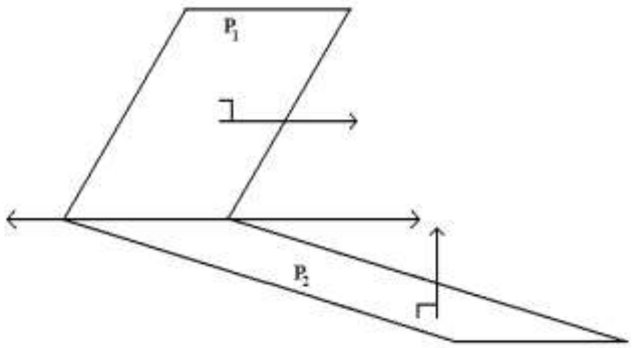
	$(x^3+x^2+x+1)\frac{dy}{dx}=2x^2+x$ $\frac{dy}{dx}=\frac{2x^2+x}{x^3+x^2+x+1}$ $\frac{dy}{dx}=\frac{2x^2+x}{(x+1)\,(x^2+1)}$ $\frac{2x^2+x}{(x+1)\,(x^2+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+1}$ $2x^2+x=A(x^2+1)+(Bx+C)(x+1)$ $=Ax^2+A+Bx^2+Bx+Cx+C$ $=(A+B)x^2+(B+C)x+(A+C)$ A + B = 2 ———————— (1) B + C = 1 ———————— (2) B = $^3/_2$, A = ½, C = – ½ from (1) & (iv) 2B = 3 A + C = 0 ———————— (iii) B – A = 1 from (2) & (iii) ———————— (iv) $\frac{dy}{dx}=\frac{½}{x+1}+\frac{½(3x-1)}{x^2+1}$ $dy=\frac{1}{2}\left[\frac{dx}{x+1}+\frac{3x-1}{x^2+1}\right]dx$ $dy=\frac{1}{2}\left[\frac{dx}{x+1}+\frac{3}{2}.\frac{2x}{x^2+1}-\frac{1}{x^2+1}\right]dx$ $dy=\frac{1}{2}\int\frac{dx}{x+1}+\frac{3}{2}\int\frac{x\,dx}{x^2+1}-\frac{1}{2}\int\frac{dx}{x^2+1}$ $y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log(x^2+1)-\frac{1}{2}\tan^{-1}x+c$ $y=\frac{1}{4}\left[\log(x+1)^2(x^2+1)^3\right]-\frac{1}{2}\tan^{-1}x+c$	1 1 1 1
20.	$\vec{a}.\vec{b}=0$ and $\vec{a}.\vec{c}=0 \Rightarrow \vec{a}\perp\vec{b}$ and $\vec{a}\perp\vec{c}$ $\therefore \vec{a}$ is $\perp$ to the plane of $\vec{b}$ and $\vec{c}$ $\Rightarrow \vec{a}$ is parallel to $\vec{b}\times\vec{c}$ Let $\vec{a}=k(\vec{b}\times\vec{c})$, where k is a scalar $\therefore \lvert\vec{a}\rvert=\lvert k\rvert\,\lvert(\vec{b}\times\vec{c})\rvert$ $1=\lvert k\rvert\;\lvert\vec{b}\rvert\,\lvert\vec{c}\rvert\sin\frac{\pi}{6}$	1 1

	$1=\|k\|\ .1.1\ .\frac{1}{2}$ $\|k\|=2 \Rightarrow k=\pm 2$ $\therefore \vec{a}=\pm(\vec{b}\times\vec{c})$	1 1
21.	As $2:3:4 \neq 5:2:1$, the lines are not parallel Any point on the first line is $(2\lambda+1, 3\lambda+2, 4\lambda+a)$ Any point on the second line is $(5\mu+4, 2\mu+1, \mu)$ Lines will be skew, if, apart from being non parallel, they do not intersect. There must not exist a pair of values of λ, μ, which satisfy the three equations simultaneously: $2\lambda+1=5\mu+4, 3\lambda+2=2\mu+1, 4\lambda+a=\mu$Solving the first two equations, we get $\lambda=-1, \mu=-1$ These values will not satisfy the third equation if $a \neq 3$	½ ½ ½ ½ 1/2 ½ 1
22.	X = 0, 1, 2, 3 P(X) = $P(X)=\left(\frac{5}{6}\right)^3, \left(\frac{5}{6}\right)^2.\left(\frac{1}{6}\right), \left(\frac{5}{6}\right).\left(\frac{1}{6}\right), \left(\frac{1}{6}\right)$ $=-50000\times 3+(-50000\times 2+100000)+(-50000+100000)+100000$ $=-150000+150000$ $=0$ E(X) = $E(X)=\left(\frac{5}{6}\right)^3\times 3+\left(\frac{5}{6}\right)^2.\left(\frac{1}{6}\right)\times 2+\left(\frac{5}{6}\right).\left(\frac{1}{6}\right)\times 1+\left(\frac{1}{6}\right)\times 0=\frac{455}{216}$	½ ½ 1 ½ 1+1/2
23.	Let event E_1, E_2, E_3, E_4 and A are as follow E_1 : Missing card is a diamond E_2 : Missing card is a spade E_3 : Missing card is a club E_4 : Missing card is a heart A : Drawing two spade cards P (E_1) = P (E_2) = P (E_3) = P (E_4) = $\frac{1}{4}$ P (A/E_2) = $\frac{12}{51}\times\frac{11}{50}$ P (A/E_3) = P (A/E_1) = P (A/E_4) = $\frac{13}{51}\times\frac{12}{50}$	½ 1/2 ½ ½

	P (E$_3$/A) = $\dfrac{P(E_4)\ P\left(\dfrac{A}{E_4}\right)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)+P(E_4)P(A/E_4)}$	1/2
	$=\dfrac{\frac{1}{4}\times\frac{13}{51}\times\frac{12}{50}}{\frac{1}{4}\times\left(\frac{12\times11+13\times12+13\times12+13\times12}{51\times50}\right)}$	1
	$=\dfrac{13\times12}{12\times3\times13+2\times11}=\dfrac{13}{39+11}=\dfrac{13}{50}$	1/2
	Section-D	
24.	$Let\ I=\int\sqrt[3]{\tan x}\ dx$ $put\ \tan x=t^3\Rightarrow \sec^2 x dx=3t^2dt$ $(1+t^6)dx=3t^2dt$	1
	$I=\int\dfrac{t.3t^2dt}{(1+t^6)}$ $=\int\dfrac{t.3t^2dt}{\{1+(t^2)^3\}} Now\ Put\ t^2=z\Rightarrow 2t\,dt=dz$ $=\dfrac{3}{2}\int\dfrac{z\ dz}{1+z^3}$ $=\dfrac{3}{2}\int\dfrac{z\ dz}{(1+z)(1-z+z^2)}$	1
	$Now\dfrac{z}{(1+z)(1-z+z^2)}=\dfrac{A}{(1+z)}+\dfrac{Bz+C}{(1-z+z^2)}$ $Now\ \ z=A(1-z+z^2)+(Bz+C)(1+z)......(1)$ putting z = -1 in (1) -1=3A ⇒ A= -1/3 by equating the coefficient we get B=1/3=C	1
	$Now\dfrac{3}{2}\int\left[\dfrac{-1/3}{(1+z)}+\dfrac{1/3z+1/3}{(1-z+z^2)}\right]dz$ $=\dfrac{-1}{2}\int\left[\dfrac{1}{(1+z)}-\dfrac{\frac{1}{2}(2z-1)+\frac{3}{2}}{(1-z+z^2)}\right]dz$	1

	$= \frac{-1}{2}\int\left[\frac{1}{(1+z)} - \frac{\frac{1}{2}(2z-1)}{(1-z+z^2)} - \frac{3}{2}\frac{1}{(1-z+z^2)}\right]dz$	
	$= \frac{-1}{2}\int\left[\frac{1}{(1+z)} - \frac{\frac{1}{2}(2z-1)}{(1-z+z^2)} - \frac{3}{2}\frac{1}{\left(z-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\right]dz$	1
	$= \frac{-1}{2}\left[\log(1+z) - \frac{1}{2}\log(1-z+z^2) - \frac{3}{\not 2}\frac{\not 2}{\sqrt{3}}\tan^{-1}\frac{z-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right] + C$	
	$I = \frac{-1}{2}\left[\log\left\{1+\left(\tan^{2/3}x\right)\right\} - \frac{1}{2}\log\left\{1-\left(\tan^{2/3}x\right)+\left(\tan^{2/3}x\right)^2\right\} - \sqrt{3}\tan^{-1}\frac{2\left(\tan^{2/3}x\right)-1}{\sqrt{3}}\right] + C$	1
	OR	
	$\int_1^4 (x^2-x)\,dx \qquad nh = 4-1 = 3$	
	$\int_a^b f(x) = \lim_{h\to 0} h[f(a) + f(a+h) + \ldots + f\{a+(n-1)h\}]$ **where h > 0 (however small)**	
	$\int_1^4 f(x) = \lim_{h\to 0} h[f(1) + f(1+h) + \ldots + f\{1+(n-1)h\}]$	
	$= \lim_{h\to 0} h\left[(1^2-1) + \{1+h^2+2h-1-h\} + \ldots + (1+(n-1)h)^2 - (1+h-1)h\right]$	
	$= \lim_{h\to 0} h\left[(1^2-1) + \{(1+h)^2-(1+h)\} + \ldots\{1+(n-1)^2h^2+2(n-1)h-1-(n-1)h\}\right]$	
	$= \lim_{h\to 0} h\left[0 + (h^2+n) + \ldots + \{(n-1)^2 + (n-1)h\}\right]$	
	$= \lim_{h\to 0} h\left[\{h^2 + (2h)^2 + (3h)^2 + \ldots + (n-1)h^2\} + \{h + 2h + - + (h-1)h\}\right]$	
	$= \lim_{h\to 0} h\left[\{h^2\left(1^2+2^2+3^2+\ldots+(n-1)^2\right)\} + h\{1+2+3\ldots+(n-1)\}\right]$	
	$= \lim_{h\to 0} h\left[h^2\frac{n(n-1)(2n-1)}{6} + \frac{nh(nh-h)}{2}\right]$	
	$= \lim_{h\to 0} h\left[\frac{nh(nh-h)(2nh-h)}{6} + \frac{nh(nh-h)}{2}\right]$	
	$= \frac{3(3-0)(6-0)}{6} + \frac{3(3-0)}{2}$	

	$= 9 + \frac{9}{2} = \frac{27}{2}$	
25.	So, $*$ is commutative on A (ii) $*$ is commutative $(a, b), (c, d) \notin N \times N$, we have $(a, b) * (c, d) = (ad + bc, bd)$ $(c, d) * (a, b) = (cb + da, db)$ $\therefore$ addition and multiplication are commutative on N therefore $ad + bc = cb + da$ and $bd = db$ $\Rightarrow (a, b) * (c, d) = (c, d) * (a, b)$ (i) Let (x, y) be the identity element in A $(a, b) * (x, y) = (a, b) \; \forall a, b \in N$ $(ay + bx, by) = (a, b) \; \forall a, b \in N$ $a y + b x = a$ by $= b$ $x = 0, \; y = 1$ but $0 \notin N$ therefore $(0, 1) \notin N \times N$ $\therefore$ there is no identity element in A with respect of $*$.	
26.	Ordinate of intersection points are 1 & – 2 therefore Required area  = area ABCD A – *ar* ABEA $= \int_{-2}^{1} (2 - y) dy - \int_{-2}^{1} y^2 \, dy$	2+1/2 ½

	$= \left(2 - \frac{y^2}{2}\right)_{-2}^{1} - \left[\frac{y^3}{3}\right]_{-2}^{1}$ $= \left(2-\frac{1}{2}\right) - \left\{-4-\frac{4}{2}\right\} - \left(\frac{1}{3} - \frac{-8}{3}\right)$ $= \frac{3}{2}+6-3$ $= \frac{3}{2}+3 = 4.5$ (sq. units) **OR**  Required Area is the region lying Shaded region = $\int_2^3 y(parabola)\,dx - \int_2^3 y(line)\,dx = \int_2^3 (x^2+3)\,dx - \int_2^3 (2x+3)\,dx$ $= \int_2^3 (x^2)\,dx - \int_2^3 (2x)\,dx = \left[\frac{x^3}{3}\right]_2^3 - \left[x^2\right]_2^3$ $= \frac{4}{3}\, sq.units$	1 1 1 Fig.2.5 ½ 2 1
27.	 We have plant P_1 & P_2 $4x + 4y - 5z = 12$ $8x + 12y - 13z = 32$ Let a, b, c are the direction ratio of vector | | to the line. $\therefore \quad 4a+4b-5c=0$ $8a+12b-13c=0$ $\frac{a}{-52+60} = \frac{-b}{-52+40} = \frac{c}{48-32}$	

$$\frac{a}{8}=\frac{b}{12}=\frac{c}{16}$$

$$\frac{a}{2}=\frac{b}{3}=\frac{c}{4}$$

Here 4 $\neq$ 0 therefore line of intersection is not parallel to *xy* – plane.

Let the line of intersection meet the *xy* – plane at P ($\propto$, β , o).

Then P lies on plane (1) & (2)

$\therefore$ $4\propto +4\beta-12=0$

$\propto+\beta-3=0$ ———— (ii)

& $8\propto +12\beta-32=0$

$2\propto +3\beta-8=0$ ———— (1)

On solving (1) & (ii) we get

$\propto=1 \quad \beta=2$

Hence eq^n of line,

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-0}{4}$$

28.	Given equation $\Rightarrow \frac{1}{pq}\begin{vmatrix} pq & q^2 & pq\alpha+q^2 \\ pq & pr & pq\alpha+pr \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix}=0$	1
	$\Rightarrow \frac{1}{pq}\begin{vmatrix} 0 & q^2-pr & q^2-pr \\ pq & pr & pq\alpha+pr \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix}=0 \ (R_1 \to R_1-R_2)$	1
	$\Rightarrow \frac{q^2-pr}{pq}\begin{vmatrix} 0 & 1 & 1 \\ pq & pr & pq\alpha+pr \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix}=0$	1
	$\Rightarrow \frac{q^2-pr}{pq}p\begin{vmatrix} 0 & 1 & 1 \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix}=0$	1
	$\Rightarrow \frac{q^2-pr}{q}\begin{vmatrix} 0 & 0 & 1 \\ q & -q\alpha & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix}=0(C_2 \to C_2-C_3)$	
	$\Rightarrow \frac{q^2-pr}{q}(q^2\alpha+rq+pq\alpha^2+q^2\alpha)=0 \Rightarrow (q^2-pr)(2q\alpha+r+p\alpha^2)=0 \Rightarrow q^2-pr=0$ (i.e., p, q, r	1
	are in GP) or $2q\alpha+r+p\alpha^2$ =0(i.e., α is a root of the equation $2qx+r+px^2$ =0	1
	OR	
		1

$$A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$$ **1**

$$=\begin{bmatrix}1+1+2 & 1+2-1 & 1-3+3\\ 1+2-6 & 1+4+3 & 1-6-9\\ 2-1+6 & 2-2-3 & 2+3+9\end{bmatrix}=\begin{bmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{bmatrix}$$ **1**

$$A^3=A^2\cdot A=\begin{bmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}4+2+2 & 4+4-1 & 4-6+3\\ -3+8-28 & -3+16+14 & -3-24-42\\ 7-3+28 & 7-6-14 & 7+9+42\end{bmatrix}$$

$$=\begin{bmatrix}8 & 7 & 1\\ -23 & 27 & -69\\ 32 & -13 & 58\end{bmatrix}$$

1

1

$\therefore A^3-6A^2+5A+11I$

$$=\begin{bmatrix}8&7&1\\-23&27&-69\\32&-13&58\end{bmatrix}-6\begin{bmatrix}4&2&1\\-3&8&-14\\7&-3&14\end{bmatrix}+5\begin{bmatrix}1&1&1\\1&2&-3\\2&-1&3\end{bmatrix}+11\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}8&7&1\\-23&27&-69\\32&-13&58\end{bmatrix}-\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}+\begin{bmatrix}5&5&5\\5&10&-15\\10&-5&15\end{bmatrix}+\begin{bmatrix}11&0&0\\0&11&0\\0&0&11\end{bmatrix}$$

$$=\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}-\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}$$ **1**

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$$

Thus, $A^3-6A^2+5A+11I=O$.

Now,

$A^3-6A^2+5A+11I=O$

$\Rightarrow (AAA)A^{-1}-6(AA)A^{-1}+5AA^{-1}+11IA^{-1}=0$ $\quad$ $\left[\text{Post-multiplying by } A^{-1} \text{ as } |A|\neq 0\right]$

$\Rightarrow AA(AA^{-1})-6A(AA^{-1})+5(AA^{-1})=-11(IA^{-1})$

$\Rightarrow A^2-6A+5I=-11A^{-1}$

$\Rightarrow A^{-1}=-\frac{1}{11}(A^2-6A+5I) \quad \ldots(1)$

Now,

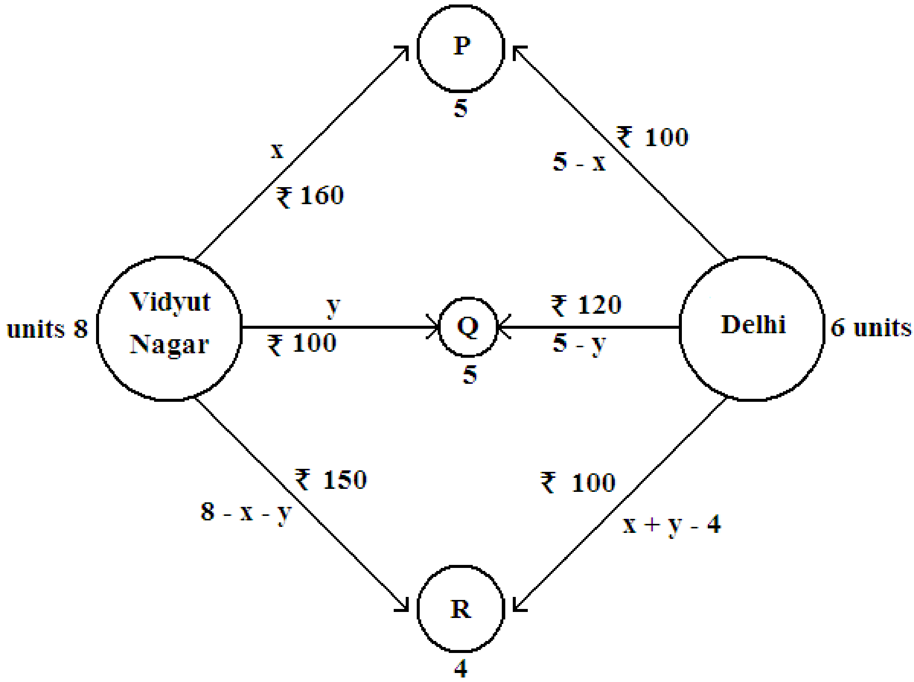
$A^2-6A+5I$

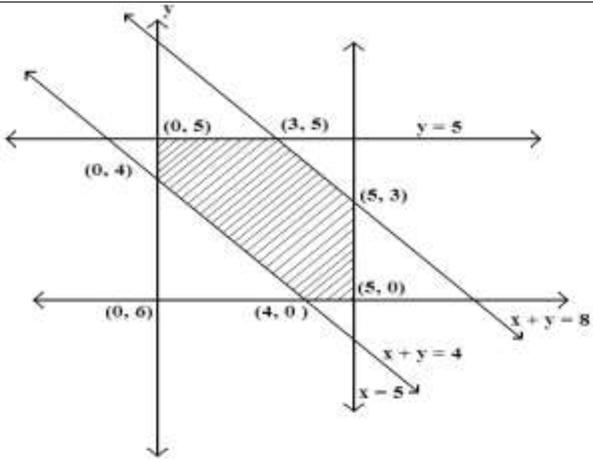
$$=\begin{bmatrix}4&2&1\\-3&8&-14\\7&-3&14\end{bmatrix}-6\begin{bmatrix}1&1&1\\1&2&-3\\2&-1&3\end{bmatrix}+5\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}4&2&1\\-3&8&-14\\7&-3&14\end{bmatrix}-\begin{bmatrix}6&6&6\\6&12&-18\\12&-6&18\end{bmatrix}+\begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}$$

$$=\begin{bmatrix}9&2&1\\-3&13&-14\\7&-3&19\end{bmatrix}-\begin{bmatrix}6&6&6\\6&12&-18\\12&-6&18\end{bmatrix}$$

$$=\begin{bmatrix}3&-4&-5\\-9&1&4\\-5&3&1\end{bmatrix}$$

	From equation (1), we have: $A^{-1} = -\frac{1}{11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$	
29.	Let x units and y units of the commodity be transported from the factory at Vidyut Nagar to the depots the P and Q respectively. Then $8 - x - y$ units will be transported to depots C. 	½
	L PP is Minimize $z = 10x - 70y + 1900$ Subject to constraints $x \geq 0, \quad y \geq 0$ ----------- (i) $x + y \leq 8$ ----------- (ii) $x \leq 5$ ----------- (iii) $y \leq 5$ ----------- (iv) $x + y \geq 4$ ----------- (v)	**1+1/2**
	Correct graph :	**2**



1

Observed that feasible region is bounded coordinate of corner points of the feasible regions are (0, 4), (0, 5), (3, 5), (5, 3), (5, 0) and (4, 0) Let us evaluate z at these points.

Hence the optimal transportation strategy will be to deliver 0, 5 and 3 units from Vidyut Nagar and 5, 0 and 1 from Delhi to the depots P, Q & R respectively. Corresponding to this strategy the min transportation cost is ` 1500. **1**

Corner points

(0, 4)

(0, 5)

(3, 5)

(5, 3)

(5, 0)

(4, 0)

z = 10 (*x* – 7*y* + 190)

1620

1550 → Min

1580

1740

1950

1940

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Submitted by S S Shishodia PGT (Maths), DPS Jodhpur (Raj.) ph.9887701111